

Archdiocese of Westminster

**St. Joseph's Catholic Primary School
Hertford**



'Together we grow in God's love, learning to be the best we can be'

Calculation Policy

May 2015

Ite ad Joseph

CALCULATION POLICY

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the National Curriculum commencing September 2014.

Through the policy, we aim to link key manipulatives and representations in order that the pupils can be vertically accelerated through each strand of calculation.

We know that school wide policies, such as this, can ensure consistency of approach, enabling pupils to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency.

As pupils move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014; however it is vital that pupils are taught according to the stage that they are currently working at, being moved onto the next level as soon as they are secure enough to move on.

Real life context:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

To add and subtract successfully, children should be able to:

- recall all addition pairs to $9 + 9$ and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. $5 + 8 + 4$)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. $600 + 700$, $160 - 70$)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to $12 \times 12 = 144$ and division facts to $144 \div 12 = 12$
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that $15 = 3 \times 5$, or that $40 = 10 \times 4$) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings

- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

Progression in addition and subtraction

Addition and subtraction are connected.

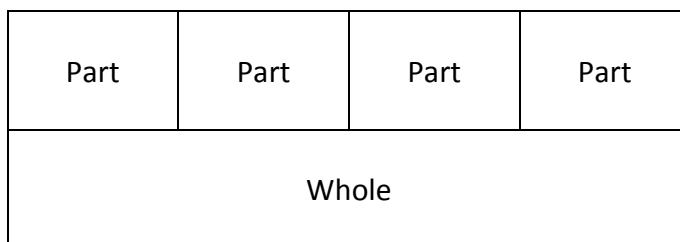
Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

<u>Gradation of difficulty- addition:</u>	<u>Gradation of difficulty- subtraction:</u>
1. No exchange	1. No exchange
2. Extra digit in the answer	2. Fewer digits in the answer
3. Exchanging ones to tens	3. Exchanging tens for ones
4. Exchanging tens to hundreds	4. Exchanging hundreds for tens
5. Exchanging ones to tens and tens to hundreds	5. Exchanging hundreds to tens and tens to ones
6. More than two numbers in calculation	6. As 5 but with different number of digits
7. As 6 but with different number of digits	7. Decimals up to 2 decimal places (same number of decimal places)
8. Decimals up to 2 decimal places (same number of decimal places)	8. Subtract two or more decimals with a range of decimal places
9. Add two or more decimals with a range of decimal places	

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.



<u>Gradation of difficulty (short multiplication)</u>	<u>Gradation of difficulty (short division)</u>
<ol style="list-style-type: none"> 1. TO x O no exchange 2. TO x O extra digit in the answer 3. TO x O with exchange of ones into tens 4. HTO x O no exchange 5. HTO x O with exchange of ones into tens 6. HTO x O with exchange of tens into hundreds 7. HTO x O with exchange of ones into tens and tens into hundreds 8. As 4-7 but with greater number digits x O 9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O 	<ol style="list-style-type: none"> 1. TO ÷ O no exchange no remainder 2. TO ÷ O no exchange with remainder 3. TO ÷ O with exchange no remainder 4. TO ÷ O with exchange, with remainder 5. Zero in the quotient e.g. $816 \div 4 = \mathbf{204}$ 6. As 1-5 HTO ÷ O 7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. $7.5 \div 5$ or $0.12 \div 3$ 9. Where the divisor is a two digit number <p>See below for gradation of difficulty with remainders</p>
<p><u>Long multiplication—multiplying by more than one digit</u> Children will refer back to grid method by using place value counters or Base 10 equipment with no exchange and using synchronised modelling of written recording as a long multiplication model before moving to TO x TO etc.</p>	<p><u>Long division —dividing by more than one digit</u> Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as either:-</p> <ol style="list-style-type: none"> 1. Chunking model of long division using Base 10 equipment 2. Sharing model of long division using place value counters <p>See the following pages for exemplification of these methods.</p>

+ Addition +

Foundation Stage

Children will engage in a wide variety of songs and rhymes, games and activities.

They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number. They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.

Progression of calculation in Foundation Stage

U+U (aggregation: combining objects)

1 more than a given number up to 20.

U+U (augmentation: counting on)

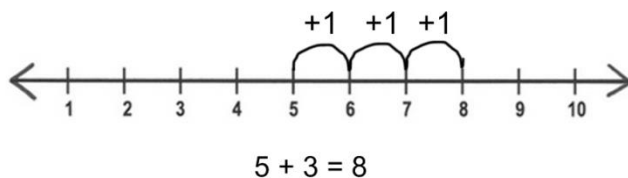
Written methods

- Make a record in counters, pictures or marks of addition activities.

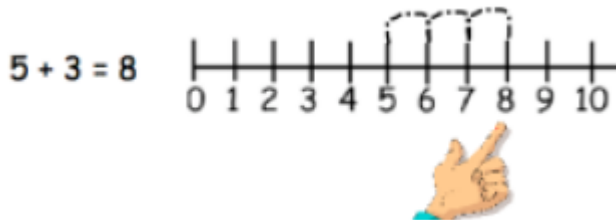


‘You have five apples and I have three apples. How many apples altogether?’

Teacher demonstrates how to record on a number line and using numbers and symbols including + and =



Children jump forwards along the number line using a finger and, if ready, record using number lines and symbols.



Expectations:

Can count reliably to at least 20.

Can find one more than a given number.

Can estimate how many objects they can see and check by counting.

Use the language ‘more’ to compare two sets of objects.

+ Addition +

Year 1

Mental Calculations

- Read, write and interpret mathematical statements using symbols +, -, =
- Represent and use number bonds and related addition facts within 20
- Add one digit and two-digit numbers up to 20, including zero.
- Solve one-step problems using concrete objects and pictorial representations, and missing number problems such as $6 = \square - 9$
- Given a number, identify (and use the language) one more

Written Calculations

- Begin to compare (what's the same/different?) for commutative sums e.g $3 + 7 = 7 + 3$
- Memorise and reason with number bonds to 10 & 20 in several forms
- Add using objects, dienes, cubes etc and number lines and tracks
- Check with everyday objects
- Ensure pre-calculation steps are understood, including:
 - Counting objects (including solving simple concrete problems)
 - Conservation of number:
 - Recognise place value in numbers beyond 20

Counting as reciting and as enumerating

Add with numbers up to 20

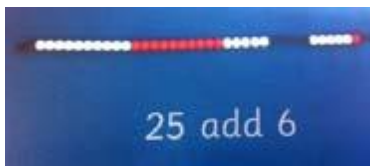
$$8 + 7 = 15$$

'Put your finger on number eight and count on seven.'

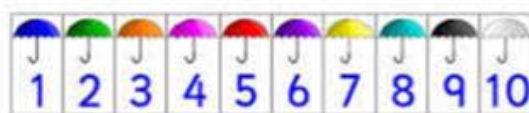


Representations to support mental and written calculations

Use a range of concrete and pictorial representations, including:



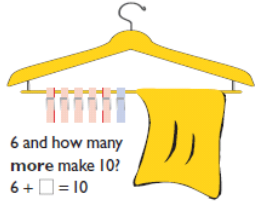
Bead strings



Number lines



Which line has most money?
How much more?



6 and how many
more make 10?
 $6 + \square = 10$



Links to the curriculum

- ✓ Read and write numbers to 100 in numerals, incl. 1—20 in words
- ✓ Recall bonds to 10 and 20, and addition facts within 20
- ✓ Count to and across 100
- ✓ Count in multiples of 1 2, 5 and 10
- ✓ Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line.

Video explanation

Number Sense and place value in Key Stage 1

[Counting in steps of one and ten](#)

Addition and subtraction in Key stage 1

[Partitioning in different ways](#)

[Using resources to develop fluency and understanding](#)

[Number bonds to 10](#)

[Consolidation and practice](#)

+ Addition + Year 2

Mental and Written Calculations

Add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers
- Recall and use addition and subtraction facts to 20 facts fluently, and derive and use related facts up to 100
- Demonstrate the commutative law of addition
- Re-partition numbers eg.
- Use a hundred square
- Check calculations using inverse and by adding numbers in different order
- Begin to record addition in columns to support place value and prepare for formal written methods with larger numbers

$$17 + 2 = 19 \quad 12 + 4 = 16$$

$$57 + 2 = 59 \quad 32 + 34 = 66$$

$$12 + 30 = 30 + 12$$

$$\square + 25 = 25 + 41$$

$65 = 60 + 5$
$65 = 50 + 15$
$65 = 40 + 25$
$65 = 30 + 35$
$65 = 20 + 45$
$65 = 10 + 55$

$$30 + 4$$

$$\underline{20 + 5}$$

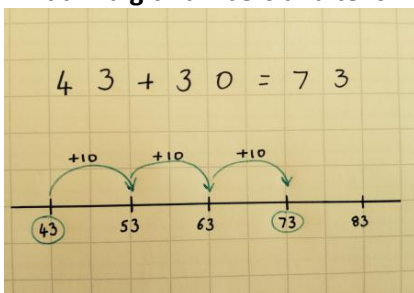
$$50 + 9$$

Add with 2-digit numbers

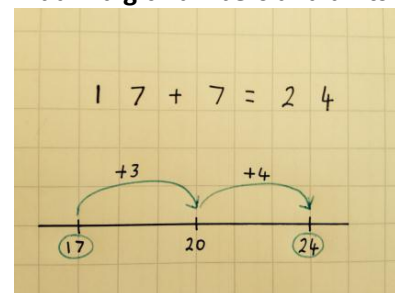
Developing mental fluency with addition and place value involving 2-digit numbers, then progressing to more formal methods.

Phase 1

Add 2-digit numbers and tens:



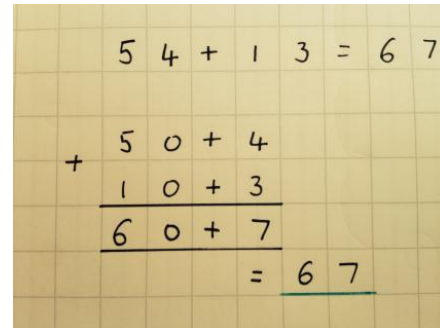
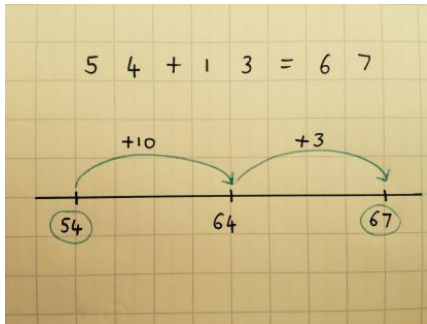
Add 2-digit numbers and units:



Use empty number lines and hundred squares to build confidence and fluency in mental addition skills.

Phase 2

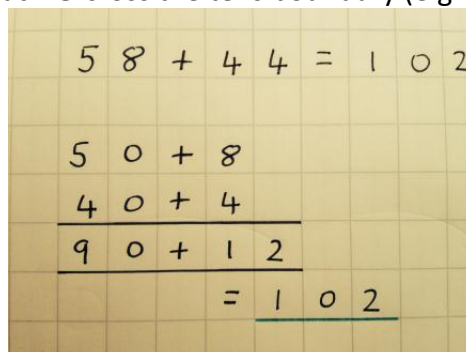
Add pairs of 2-digit numbers, progressing to the partitioned column method when secure adding tens and units:



Addition at this stage should **NOT** cross the tens boundary until the children are secure with the method itself.

Phase 3

Once children can add a multiple of ten to a 2-digit number mentally (e.g. $60+12$), they are ready to start adding pairs of 2-digit numbers that **DO** cross the tens boundary (e.g. $68 + 33$).



Children who are confident and accurate with this phase should move onto the expanded addition methods with 2 and 3-digit numbers (Y3).

Links to the curriculum:

- ✓ Add a 2-digit number and ones (e.g. $27 + 6$)
- ✓ Add a 2-digit number and tens (e.g. $23 + 40$)
- ✓ Add pairs of 2-digit numbers (e.g. $35 + 47$)
- ✓ Add three single-digit numbers (e.g. $5 + 9 + 7$)
- ✓ Show that adding can be done in any order (the commutative law).
- ✓ Recall bonds to 20 and bonds of tens to 100 ($30 + 70$ etc.)
- ✓ Count in steps of 2, 3 and 5 and count in tens from any number.
- ✓ Understand the place value of 2-digit numbers (tens and ones)
- ✓ Compare and order numbers to 100 using $<$ $>$ and $=$ signs.
- ✓ Read and write numbers to at least 100 in numerals and words.
- ✓ Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary.

+ Addition +

Year 3

Mental Calculations

Add numbers mentally, including:

- a three-digit number and ones
- a three-digit number and tens
- a three digit number and hundreds
 - Partition all numbers and recombine, start with TU + TU then HTU + TU
 - Use straws, dienes, place value counters, empty number lines

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging though ten, hundred
- Complementary addition

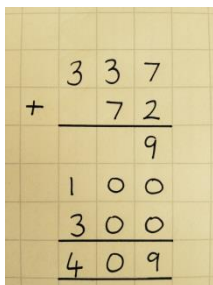
Written Calculations

Add numbers with up to 3 digits

The **expanded column addition** method:

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.



	3	3	7
+		7	2
			9
	1	0	0
	3	0	0
	4	0	9

Always start by adding the units first, followed by the tens and then finally the hundreds.



The **compact column addition** method, with 'carrying':

Children should be made aware of the reduced number of steps required for this method. However, they should also be reminded that the actual value is **'four tens add seven tens'** not **'four add seven'**

$$\begin{array}{r} 443 \\ + 75 \\ \hline 518 \\ 1 \end{array}$$

Add the ones first.

Always 'carry' numbers underneath the bottom line.

Links to the curriculum:

- ✓ Read and write numbers to 1000 in numerals and words.
- ✓ Add 2-digit numbers mentally, incl. those exceeding 100.
- ✓ **Add a three-digit number and ones mentally (175 + 8)**
- ✓ **Add a three-digit number and tens mentally (249 + 50)**
- ✓ **Add a three-digit number and hundreds mentally (381 + 400)**
- ✓ Estimate answers to calculations, using inverse to check answers.
- ✓ Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- ✓ Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- ✓ Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Keyvocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact.

+ Addition +

Year 4

Mental Calculations

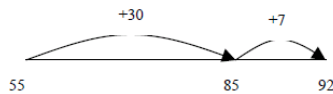
Practise mental methods with increasingly large numbers

Consolidate partitioning and re-partitioning

Use compensation for adding too much/little and adjusting

Use straws, Dienes, place value counters, empty number lines etc.

I know that $63 + 29$ is the same as $63 + 30 - 1$



$$\begin{aligned} 55 + 37 &= 55 + 30 + 7 \\ &= 85 + 7 \\ &= 92 \end{aligned}$$

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging through ten, hundred
- Complementary addition

Written Calculations

Add numbers with up to 4 digits

In Year 4 the children should progress from expanded addition to the compact column method. It is important that **the units are always added first** and when 'carrying' numbers the carried digits are always placed **underneath** the calculation. The children will also be introduced to adding in the contexts of money and measures.

Always 'carry' numbers underneath the bottom line.

$789 + 642$ becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline \end{array}$$

Answer: 1431

Add the units first.

Children should be reminded that the actual value is '**seven hundred** add **six hundreds**' not '**seven** add **six**'.

Links to the curriculum:

- ✓ Select most appropriate method: mental, jottings or written and explain why.
- ✓ Recognise the place value of each digit in a four-digit number.
- ✓ Round any number to the nearest 10, 100 or 1000.
- ✓ Estimate and use inverse operations to check answers.
- ✓ Solve 2-step problems in context, deciding which operations and methods to use and why.

- ✓ Find 1000 more or less than a given number.
- ✓ Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- ✓ Add numbers with up to 4 digits using the formal written method of column addition
- ✓ Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- ✓ Estimate and use inverse operations to check answers to a calculation.

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry' expanded, compact, thousands, hundreds, digits, inverse.

+ Addition +

Year 5

Mental Calculations

- Add numbers mentally with increasingly large numbers, e.g. $12\,462 + 2300 = 14\,762$
- Mentally add tenths, and one-digit numbers and tenths
- Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of places, and complements of 1 (e.g. $0.83 + 0.17 = 1$)

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging through ten, hundred, tenth
- Complementary addition

Written Calculations

Add numbers with more than 4 digits

(including money, measures and decimals with different numbers of decimal places).

$$\begin{array}{r} + \text{£ } 34.68 \\ \text{£ } 8.46 \\ \hline \text{£ } 43.14 \\ \phantom{\text{£ }} 1 \end{array}$$

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

Children should have an understanding of the place value of **tenths and hundredths** and use this to align the digits. Numbers with different numbers of decimal places should be used.

$$\begin{array}{r} 44571 \\ + 2362 \\ \hline 46933 \\ 1 \end{array}$$

Numbers should exceed four digits

Pupils should be able to add more than two values, carefully aligning place value columns.

$$\begin{array}{r} 23.04 \\ + 5.36 \\ \hline 29.20 \\ 1 \end{array}$$

Empty decimal places can be filled with zero to show the place value in each column. Say '3 tenths add 8 tenths' to reinforce place value.

Links to the curriculum:

- ✓ Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- ✓ Use rounding to check answers and accuracy.

- ✓ Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- ✓ Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- ✓ Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- ✓ Add numbers with more than 4 digits using formal written method of columnar addition.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths.

+ Addition +

Year 6

Mental Calculations

- Perform mental calculations, including with mixed operations and large numbers (*more complex calculations*)

Children use representation of choice

Consolidate partitioning and re-partitioning

Use compensation for adding too much/little and adjusting

Common mental calculation strategies:

- Partitioning and recombining
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging though ten, hundred, tenth
- Complementary addition

Written Calculations

Add several numbers of increasing complexity

Adding several numbers with different numbers of decimal places (including money and measures):

- ✓ Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically (including in the answer row).
- ✓ Zeros could be added into any empty decimal places, to show there is no value to add.

$$\begin{array}{r} 43.471 \\ 06.040 \\ + 37.550 \\ \hline 89.361 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 73,034 \\ 4,863 \\ + 14,201 \\ \hline 122,253 \\ \hline 1111 \end{array}$$

This is an example of adding several numbers with more than 4 digits.

Links to the curriculum:

- ✓ Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.

- ✓ Solve multi-step problems in context, deciding which operations and methods to use and why.
- ✓ Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- ✓ Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- ✓ Round any whole number to a required degree of accuracy.
- ✓ Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

- Subtraction -

Foundation Stage

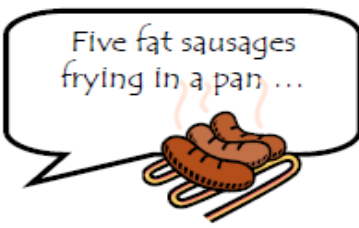
Progression of calculation

U – U (take away structure)

1 less than a given number: First from 1 – 5, moving to 1 – 10 and finally 1- 20

U – U (comparison structure, “how many more ...; “how many less ...”)

Begin to count backwards in familiar contexts such as number rhymes or stories



Five fat sausages frying in a pan ...

Ten green bottles hanging on the wall ...



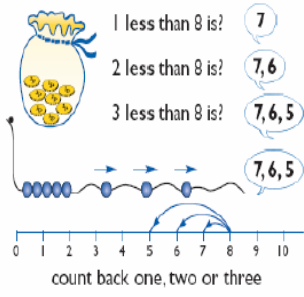
10, 9, 8, 7 ...

Continue the count back in ones from any given number

Begin to relate subtraction to 'taking away'



Three teddies take away two teddies leaves one teddy



Find one less than a number

Make a record in counters, words, or fingers of subtraction activities already carried out. Children progress to record in numbers and symbols. e.g. $6 - 3 = 3$

Children use number lines and tracks shapes to find one less and support with counting back.

Teacher demonstrates how to record counting back on a number line.

Children use objects to help them to compare objects and state how many more or less. E.g. "Six is two more than four."

Expectations:

Can find one less than a given number.

Can estimate how many objects they can see and check by counting.

Use the language 'fewer' to compare two sets of objects.

Through games, songs and practical activities children begin to use vocabulary of subtraction.

Use fingers and practical equipment to calculate.

- Subtraction -

Year 1

Mental Calculations

- Subtract one digit and two-digit numbers to 20, including zero.
- Read, write and interpret mathematical statements using symbols (+, -, =) signs.
- Represent and use number bonds and related addition facts within 20
- Solve one-step problems using concrete objects and pictorial representations, and missing number problems such as $7 = - 9$
 - Memorise and reason with number bonds
 - Add using objects, cubes etc and number lines and tracks
 - Check with everyday objects
 - Ensure pre-calculation steps are understood, including: Counting objects, Conservation of number.

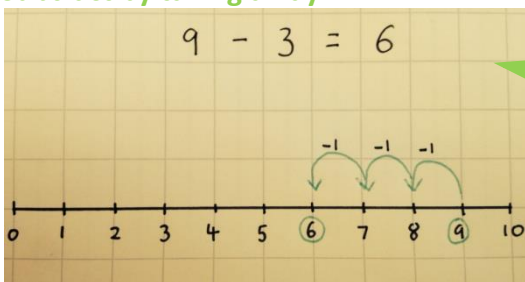
Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Written Calculations

Subtract from numbers up to 20

- Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below.
- Read, write and interpret number sentences with - and = signs.

Subtract by taking away:

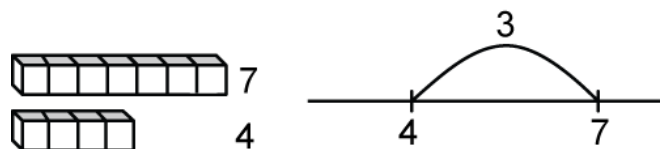


Count back in ones on a number number line to take away (with numbers up to 20).

Model subtraction using practical equipment such as 100 squares.

Subtract by finding the 'distance between':

This should be introduced practically with the language **'find the distance between'** and **'how many more?'** in a range of familiar contexts.



The difference between 7 and 4 is 3.

Links to the curriculum:

- ✓ Given a number, say **one more or one less**.
- ✓ Count to and over 100, **forward and back**, from any number.
- ✓ Represent and use **subtraction facts to 20 and within 20**.
- ✓ Subtract with **one-digit and two-digit** numbers to 20, including zero.
- ✓ Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- ✓ Read and write numbers from 0 to 20 in numerals and words.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?

- Subtraction -

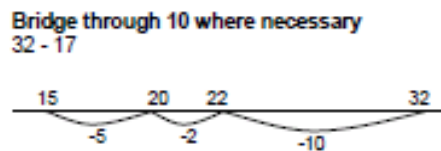
Year 2

Mental Calculations

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

Jottings to support informal methods:



Mental strategy

Subtract numbers close together by **counting on**:

Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

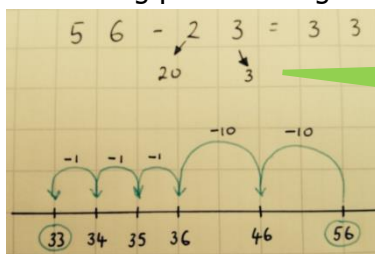
Written Calculations

Subtract with 2-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills. This area of learning consists of three phases.

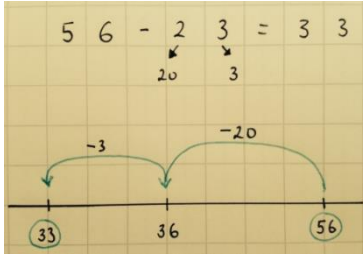
Phase 1

Subtracting pairs of 2-digit numbers on a number line:



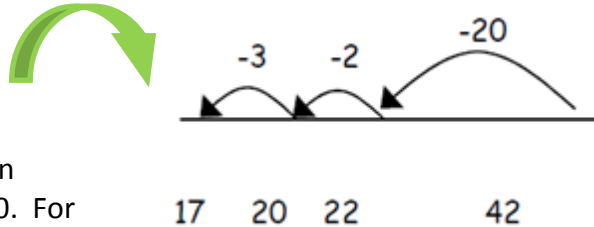
Partition the second number and subtract it in tens and ones.

Phase 2



Children should progress to more **efficient** jumps back. Combine this method with use of a hundred square to reinforce understanding of number value and order.

Phase 3



The final phase involves the children understanding how to bridge through 10. For example $42 - 25 = 17$

Links to the curriculum:

- ✓ Recognise the place value of each digit in a two-digit number.
- ✓ Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- ✓ Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- ✓ Show that subtraction of one number from another cannot be done in any order.
- ✓ Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- ✓ Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- ✓ Read and write numbers to at least 100 in numerals and in words.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, ones.

- Subtraction -

Year 3

Mental Calculations

Add and subtract numbers mentally, including:

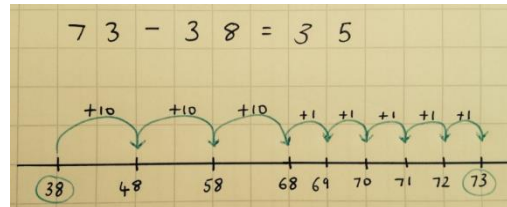
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds.

Use a number line, dienes, hundred squares, two-hundred squares, and similar representations, to support mental calculations.

Mental strategy:

As in Year 2 the 'counting on' strategy is an efficient method when finding the difference between **close-together numbers**.

The children should start at the smaller number and count on in **tens** first, then count on in ones to find the rest of the difference.



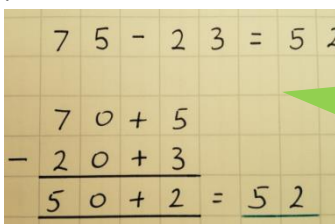
Written Calculations

Subtracting with 2 and 3-digit numbers.

The children should be introduced to the **partitioned column subtraction** method. This area of learning consists of three phases.

Phase 1

(Partitioned column subtraction- **no** exchanging required)

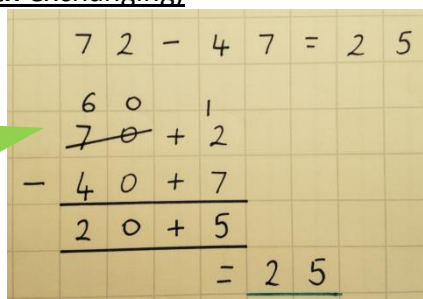


Partition the numbers and then start by subtracting the ones first.

Phase 2

(Partitioned column subtraction- **with** exchanging)

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten ones. Then subtract 7 ones, and subtract 4 tens.



$$72 - 47$$



- Subtraction -

Year 4

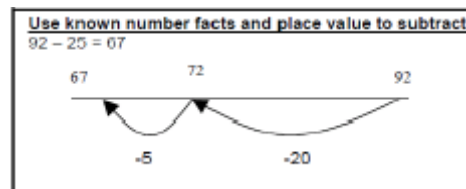
Mental Calculations

Continue to practise mental methods with increasingly large numbers to aid fluency.

Methods to support fluent calculation and encourage efficiency of method:

- Find a small difference by counting up. E.g. 5003—4996
- Subtract nearest multiple of ten and adjust.
- Partition larger numbers

This could be done using an empty number line. Children should recall and use number facts to reduce the number of steps.



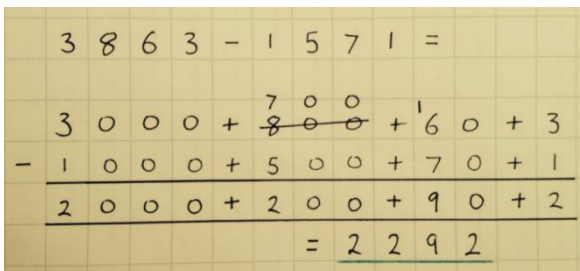
Whenever possible, children should be encouraged to visualise number lines and other basic, supporting representations to promote fluent work without jottings.

Written Calculations

Subtract with up to 4-digit numbers

Partitioned

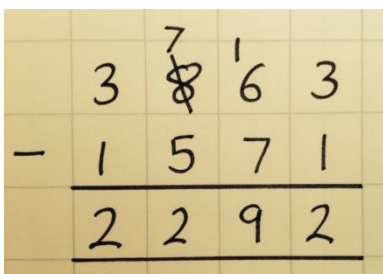
Column subtraction with 'exchanging'.



The children should establish that they are unable to take 70 away from 60 so an 'exchange' takes place between the tens and the hundreds. Once the children are secure with this method they should progress to the **compact column subtraction method**.

Compact

Column subtraction.



When introducing the compact method to the children, ask them to perform a partitioned column subtraction calculation and then display the compact subtraction method. Ask the children to distinguish the differences and similarities between the two.

This method should be applied to money and various units of measure.

Links to the curriculum:

- ✓ Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- ✓ Children select the most appropriate and efficient methods for given subtraction calculations.
- ✓ Estimate and use inverse operations to check answers.
- ✓ Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- ✓ Solve simple measure and money problems involving fractions and decimals to two decimal places.
- ✓ Find 1000 more or less than a given number.
- ✓ Count backwards through zero, including negative numbers.
- ✓ Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- ✓ Solve number and practical problems that involve the above, with increasingly large positive numbers.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse.

- Subtraction -

Year 5

Mental Calculations

- Subtract numbers mentally with increasingly large numbers.
e.g. $12\,462 - 2300 = 10\,162$
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Pupils practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, $1 - 0.17 = 0.83$).
- Pupils mentally add and subtract tenths, and one-digit whole numbers and tenths.

Basic Mental Strategies for Subtraction

- ◆ Find differences by counting up
- ◆ Partitioning
- ◆ Applying known facts
- ◆ Bridging through 10 and multiples of 10
- ◆ Subtracting 9, 11 etc. by compensating
- ◆ Counting on to, or back from the largest number

*Which method works best?
Why?
How else could we do it?*

Children use, or visualise, representation of choice.

Written Calculations

Subtract with at least 4-digit numbers
(including money, measures, decimals).

Compact column subtraction
(with larger integers and 'exchanging').

$$\begin{array}{r}
 \cancel{2}^{\text{th}} \cancel{1}^{\text{th}} \cancel{0}^{\text{th}} \cancel{8}^{\text{th}} 6 \\
 - \quad 2128 \\
 \hline
 28,928
 \end{array}$$



$$\begin{array}{r}
 \cancel{7}^{\text{th}} \cancel{1}^{\text{th}} \cancel{6}^{\text{th}} \cancel{9}^{\text{th}} \overset{8}{\cdot} 0 \\
 - \quad 372 \cdot 5 \\
 \hline
 6796 \cdot 5
 \end{array}$$

Add a 'zero' in any empty decimal places to aid understanding of what to subtract in that column.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Create lots of opportunities for subtracting and finding differences with money and measures!

Links to the curriculum:

- ✓ Subtract numbers mentally with increasingly large numbers.
- ✓ Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- ✓ Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- ✓ Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- ✓ Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- ✓ Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- ✓ Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Key vocabulary:

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

Video explanation:

Subtraction Upper Key stage 2

[Column Subtraction](#)

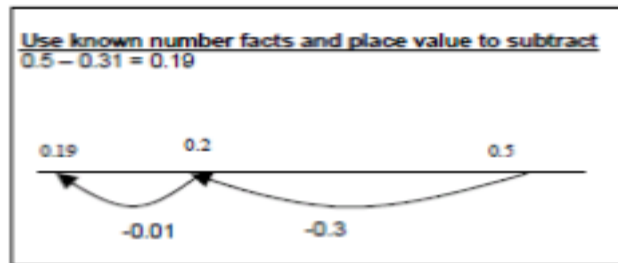
- Subtraction -

Year 6

Mental Calculations

Children:

- Perform mental calculations, including with mixed operations and large numbers.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- *They undertake mental calculations with increasingly large numbers and more complex calculations.*



Children draw on basic, mental subtraction strategies, (See Year 5.) Children use, or visualise, representation of choice.

Written Calculations

Subtracting with increasingly large and more complex numbers and decimal values.

	7	5	10	6	9	9
-	8	9	9	4	9	
	6	0	7	5	0	

Using the compact column method to subtract more complex integers



	7	10	5	·	7	1	9	kg
-	3	6	·		0	8		kg
	6	9	·		3	3	9	kg

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Links to the Curriculum:

- ✓ Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- ✓ Read, write, order and compare numbers up to 10 million and determine the value of each digit

- ✓ Round any whole number to a required degree of accuracy
- ✓ Use negative numbers in context, and calculate intervals
- ✓ across zero.
- ✓ Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

✓

Key vocabulary:

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

X Multiplication X

Foundation

Children will engage in a wide variety of songs and rhymes, games and activities.
In practical activities and through discussion they will begin to solve problems involving doubling.

Progression of calculation

Counting in 2s and 10s

Doubling single-digit numbers

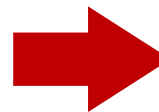
Written methods



Children use practical equipment and pictures to create sets of equal amounts together by counting.

Children make a record of doubling numbers to 5 using practical equipment, pictures and symbols.

Through songs, games, real life contexts and using practical equipment, children count in repeated groups of the same size (2s, 10s).



	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10		
11	12	13	14	15	16	17	18	19	20		
21	22	23	24	25	26	27	28	29	30		
31	32	33	34	35	36	37	38	39	40		

Children use number lines, number tracks and hundred squares to count in 2s and 10 and notice patterns.

X Multiplication X

Year 1

Mental Calculations

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Count in multiples of twos, fives and tens with equipment, songs & rhythms, and including by rote
- Counting 2s e.g. counting socks, shoes, animal legs...
- Counting in 5 s e.g. counting fingers, fingers in gloves, toes ...
- Counting in 10s e.g. counting fingers, toes ...
- Doubles up to 10
- Recognising odd and even numbers
- Write as a number pattern (e.g. 5, 10, 15...; 2, 4, 6...; 10, 20, 30...)

Written Calculations

Multiply with concrete objects, arrays and pictorial representations.

Children group quantities of small objects and make pictures to show their understanding of multiplications as adding equal sets.

e.g. $2 \times 4 = 8$; 2 groups of 4 are equal to 8.

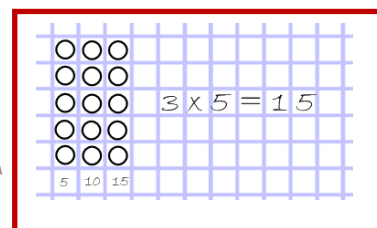


$$3 + 3 + 3 + 3 + 3 = 15$$

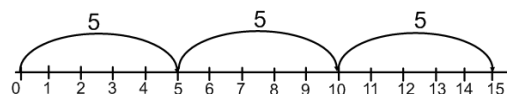
There are 3 sweets in one bag. How many sweets are in 5 bags altogether?

- Give children experience of counting equal group of objects in 2s, 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.

With support, children begin to represent multiplication sentences as arrays.



Children begin to link their knowledge of counting in 2s, 5s and 10s to multiplication situations and use a number line, number track or hundred square to help them. Teacher demonstrates using a number line and hundred-square to record multiplication of 2s, 5s and 10s.



Links to the curriculum:

- ✓ Count in multiples of 2, 5 and 10.
- ✓ Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- ✓ Make connections between arrays, number patterns, and counting in twos, fives and tens.
- ✓ Begin to understand doubling using concrete objects and pictorial representations.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

X Multiplication X

Year 2

Mental Calculations

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, connecting the 2, 5 and 10 multiplication tables to each other
- Connect the 10 multiplication table to place value
- Recognise odd and even numbers
- show that multiplication of two numbers can be done in any order (commutative)
- Use a variety of language to describe multiplication and division
- Apply doubling of numbers up to ten to doubling larger numbers

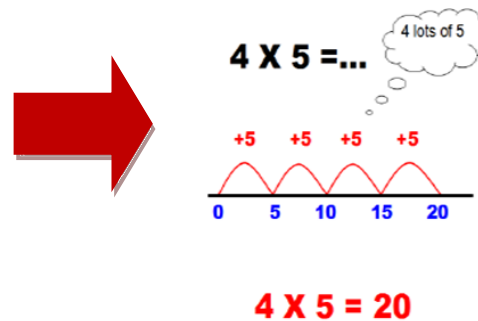
Written Calculations

Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

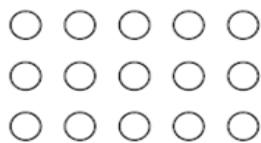
Children will show understanding of multiplication as repeated addition using different language structures. 5 times 3 , 3 groups of 5, 5 multiplied by 3 etc.

Use repeated addition on a number line:

- Children will show understanding of multiplication as repeated addition using different language structures. 4 times 5 , 4 groups of 5, 5 multiplied by 4, etc e.g. $4 \times 5 = 5 + 5 + 5 + 5$
- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.
- Children use known facts to progress on to an empty number line.



Use arrays:



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

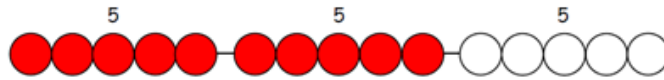
$$5 \times 3 = 3 + 3 + 3 + 3 = 15$$

$$3 \times 5 = 5 + 5 + 5 = 15$$

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times \underline{\quad} = 6$.

Use practical apparatus

$$5 \times 3 = 5 + 5 + 5$$



Links to the curriculum:

- ✓ Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- ✓ Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- ✓ Write and calculate number statements **using the x and = signs**.
- ✓ Show that multiplication can be done in any order (commutative).
- ✓ Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- ✓ Pupils use a variety of language to discuss and describe multiplication.

Key vocabulary:

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Video explanation:

Multiplication Key Stage 1

[Reinforcing table facts](#)

[Multiple Representations of Multiplication](#)

[The commutative law for multiplication](#)

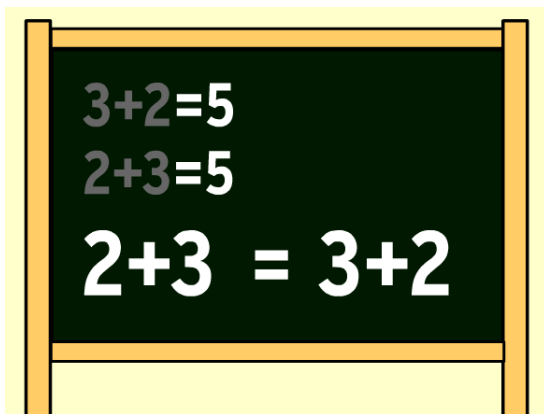
X Multiplication X

Year 3

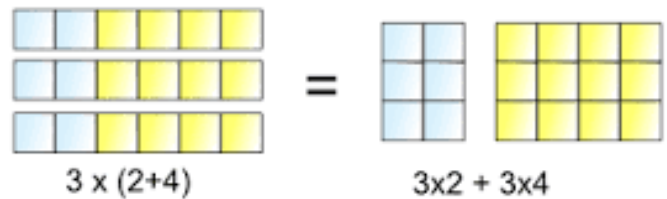
Mental Calculations

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (and 2, 5 and 10 multiplication tables)
- Use doubling to connect 2, 4 and 8 multiplication tables
- Develop efficient mental methods using commutativity and associative
- Derive related multiplication and division facts
- calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
- Partitioning: multiply the tens first and then multiply the units, e.g. $57 \times 6 = (50 \times 6) + (7 \times 6) = 300 + 42 = 342$
- Children can apply these skills to solve spoken word problems too.

The commutative law



The associative law



Ensure opportunities to learn multiplication tables through use of visual models, images and also rote learning.

Written Calculations

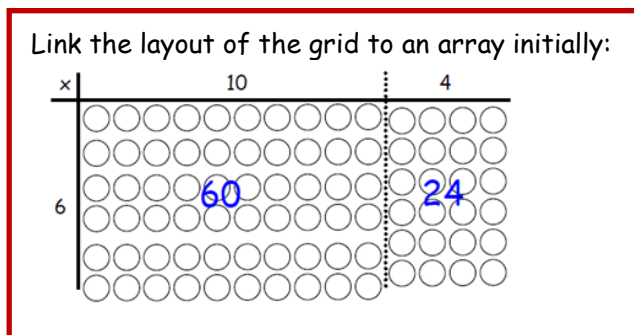
Multiply 2 -digits by a single digit number

Introduce the grid method for multiplying 2-digit by single-digits:

Eg. $23 \times 8 = 184$

X	20	3
8	160	24

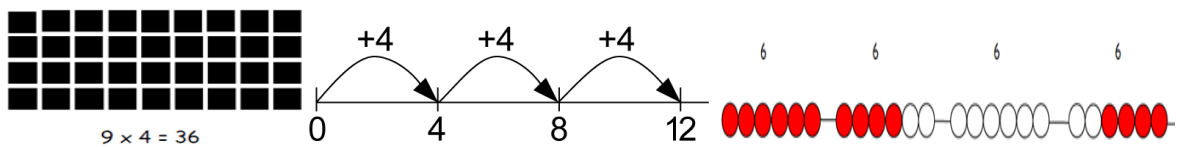
$160 + 24 = 184$



Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

To do this, children must be able to:

- ✓ Partition numbers into tens and units
- ✓ Multiply multiples of ten by a single digit (e.g. 20×4) using their knowledge of multiplication facts and place value
- ✓ Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- ✓ Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



Links to the curriculum:

- ✓ Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- ✓ Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- ✓ Solve multiplication problems, including missing number problems.
- ✓ Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)
- ✓ Solve simple problems in contexts, deciding which operations and methods to use.
- ✓ Develop efficient mental methods to solve a range of problems e.g using commutativity ($4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number problems: $? \times 5 = 20$, $3 \times ? = 18$, $? \times ? = 32$

Key vocabulary:

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value.

Video explanations:

Multiplication Lower Key Stage 2

[Grid multiplication as an interim step](#)

X Multiplication X

Year 4

Mental Calculations

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including:
 - multiplying by 0 and 1;
 - dividing by 1;
 - multiplying together three numbers
 - recognise and use factor pairs and commutativity in mental calculations
 - practise mental methods and extend this to three-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$)

Using the **distributive** law:

$$39 \times 7 = 30 \times 7 + 9 \times 7$$

Using the **associative** law:

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

Using facts and rules:

$$2 \times 6 \times 5 = 10 \times 6 = 60$$

Written Calculations

Multiply 2 and 3-digits by a single digit

(using all multiplication tables up to 12×12)

Developing the grid method:

Eg. $136 \times 5 = 680$

X	100	30	6
5	500	150	30

500
150
<u>+30</u>
<u>680</u>

Encourage column addition to add accurately.

Move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in 'carrying' for written addition.

Children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer.
e.g: 346×9 is approximately $350 \times 10 = 3500$
- Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables **up to 12 x 12**

Links to the curriculum:

- ✓ Count in multiples of 6, 7, 9, 25 and 1000
- ✓ Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- ✓ Recognise place value of digits in up to 4-digit numbers
- ✓ Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- ✓ Use commutativity and other strategies mentally $3 \times 6 = 6 \times 3$, $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$.
- ✓ Solve problems with increasingly complex multiplication in a range of contexts.
- ✓ Count in multiples of 6, 7, 9, 25 and 1000

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times, partition, grid method, total, multiple, product, sets of, inverse.

X Multiplication X

Year 5

Mental Calculations

- multiply and divide numbers mentally drawing upon known facts
- multiply and divide whole numbers and those involving decimals by 10, 100 & 1000
- Recognise and use square & cube numbers (& notation)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.

Written Calculations


Multiply up to 4-digits by 1 or 2 digits.

Introducing column multiplication

- ✓ Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method (see video).
- ✓ Children need to be taught to approximate first, e.g. for **72 x 38**, they will use **rounding: 72 x 38** is approximately $70 \times 40 = 2800$, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit:

x	300	20	7
4	1200	80	28




	3	2	7
x			4
	1	3	0
		2	8

Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits:

	10	8
10	100	80
3	30	24



	1	8
x	1	3
	5	4
	1	8
	2	3

18 x 3 on the 1st row
(8 x 3 = 24, carrying the 2 for twenty, then '1' x 3).
18 x 10 on the 2nd row. Put a zero in units first, then say 8 x 1, and 1 x 1.

Moving towards more complex numbers:

	1	2	3	4	
	×		1	6	
		7	4	0	4
1	2	3	4	0	
1	9	7	4	4	

(1234 x 6)
(1234 x 10)

Links to the curriculum:

- ✓ Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
- ✓ Solve problems where larger numbers are decomposed into their factors
- ✓ Multiply and divide integers and decimals by 10, 100 and 1000
- ✓ Recognise and use square and cube numbers and their notation
- ✓ Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times, partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'

Video explanations:

Multiplication Upper Key Stage 2

[Rapid recall of multiplication facts](#)

[Moving from grid to a column method](#)

[Demonstration of long multiplication](#)

X Multiplication X

Year 6

Mental Calculations

- perform mental calculations, including with mixed operations and large numbers (increasingly large numbers & more complex calculations).
- use all the multiplication tables to calculate mathematical statements in order to maintain fluency.
- use estimation to check answers to calculations & determine, in the context of a problem, an appropriate degree of accuracy.
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places.

Use mental strategies to solve problems e.g.

- x4 by doubling and doubling again
- x5 by x10 and halving
- x20 by x10 and doubling
- x9 by multiplying by 10 and adjusting
- x6 by multiplying by 3 and doubling

Written Calculations

Short and long multiplication as in Y5, and multiply decimals with up to 2d.p by a single digit.

Remind children that the single digit belongs in the units column.

	3	.	1	9	
x	8				
<hr/>					
2	5	.	5	2	
	1		7		

Line up the decimal points in the question and the answer

This works well for multiplying money (£.p) and other measures.

Children should be able to:

- ✓ Use rounding and place value to make approximations before calculating and use these to check answers against.
- ✓ Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit**.
- ✓ Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number**.

Links to the curriculum

- ✓ Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5)**.
- ✓ Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- ✓ Perform mental calculations with mixed operations and large numbers.
- ✓ Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- ✓ Estimate answers using round and approximation and determine levels of accuracy.
- ✓ Round any integer to a required degree of accuracy.

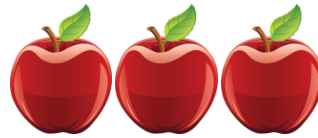
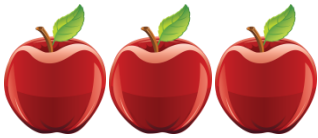
Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', tenths, hundredths, decimal.

÷ Division ÷

Foundation

Mental Calculations

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.



Share the apples between two people.
'Half of the apples for you and half of the apples for me.'

Written Calculations

Progression of calculation

- ✓ Creating equal groups of a set of objects
- ✓ Sharing a set of objects



Children use practical materials to create equal groups to solve a problem
E.g. How many cars can be made with 8 wheels?

Through opportunities which arise in the classroom, children share objects equally (this includes sharing into two sets to halve numbers).

Children will be encouraged to count out objects in 2s and 10s when grouping and sharing if appropriate.

÷ Division ÷

Year 1

Mental Calculations

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

(Pupils) make connections between arrays, number patterns, and counting in twos, fives and tens.

Children should experiment with the concepts of sharing and grouping in a number of contexts. Initially they use their own recording—moving towards fluent, symbolic notation in Year 2. Conceptual understanding and recording should be continuously supported by the use of **arrays** as a default model, as well as other representations.

The relationship between multiplication and division must be continually considered.

Written Calculations

Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

Progression of calculation

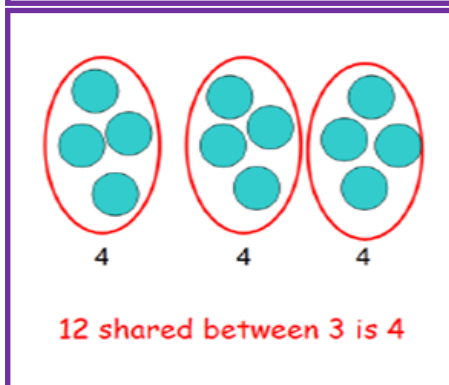
- ✓ Halving 0 – 20
- ✓ $0-20 \div u$

How many groups of 4 can be made with 12 stars? = 3

Grouping:



Sharing:



Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get? Can they work it out and give a division statement... ? "18 shared between 6 people gives you 3 each."

Pupils should:

- ✓ use lots of practical apparatus, arrays and picture representations
- ✓ Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- ✓ Be able to count in multiples of 2s, 5s and 10s.
- ✓ Find **half** of a group of objects by sharing into 2 equal groups.

Links to the curriculum:

- ✓ Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- ✓ Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- ✓ They make connections between arrays, number patterns, and counting in twos, fives and tens.

Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array.

Video explanations:

You will need to log into NCETM website to access these video clips:

[The structure of division - Key Stage 1](#)

Sharing and grouping - whole class - Video 1.1

Sharing and grouping - in pairs - Video 1.2

÷ **Division** ÷

Year 2

Mental Calculations

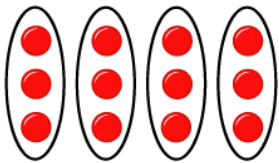
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs.

Written Calculations

Progression of Calculation

- ✓ Halving 0 – 40
- ✓ $TU \div U$ (using jottings and equipment in context)
- ✓ $TU \div 2, 5, 10$ (using known facts)
- ✓ $TU \div 2, 5, 10$ with remainders

Arrays:



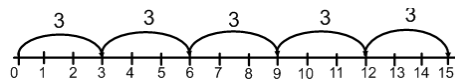
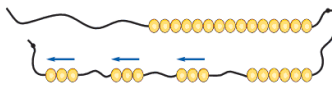
$12 \div 3 = 4$



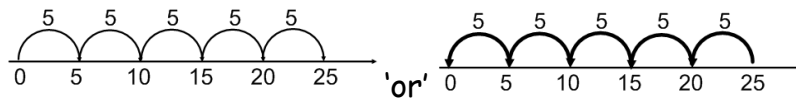
This represents **$12 \div 3$** , posed as how many groups of 3 are in 12? Pupils should also show that the same array can represent **$12 \div 4 = 3$** if grouped horizontally.

Children use bead strings and number lines to represent division as grouping. Children will be given experience of counting on in equal groups (inverse of multiplication) and counting back in equal groups (repeated subtraction).

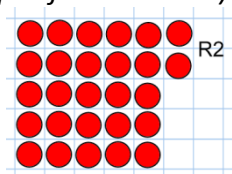
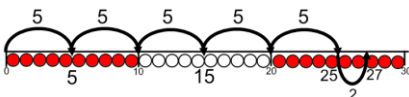
E.g. $15 \div 3 =$ (How many equal groups of 3 are in 15?)



As children learn facts, they record using an empty number line. $25 \div 5 = 5$



Children move onto calculations involving remainders with help from arrays and beaded number lines. $27 \div 5 = 5 \text{ R}2$ (How many equal groups of 5 are in 27?)



Links to the curriculum:

- ✓ Count in steps of 2, 3, and 5 from 0
- ✓ Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div and $=$ signs.
- ✓ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Key Vocabulary: share, share equally, one each, two each, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over.

÷ Division ÷

Year 3

Mental Calculations

Pupils should be taught to recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Pupils continue to practise their mental recall of multiplication tables... in order to improve fluency. Pupils develop efficient mental methods, for example, using commutativity and associativity (e.g., $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts to derive related facts.

$$\begin{array}{l} 36 \div 3 = 12 \\ 30 \\ 30 \div 3 = 10 \\ 6 \\ 6 \div 3 = 2 \end{array}$$

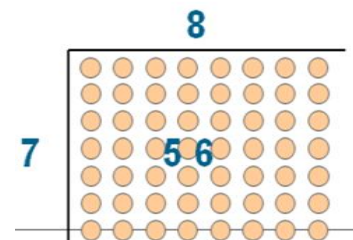


Written Calculations

Divide 2-digit numbers by a single digit (using short division).

Phase 1:

Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.



Phase 2:

Short division: Limit numbers to **NO** remainders in the answer **OR** carried (each digit must be a multiple of the divisor).

Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

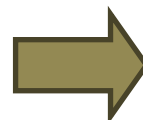
- ✓ How many 3's in 9? = 3, and record it above the **9 tens**.
- ✓ How many 3's in 6? = 2, and record it above the **6 units**



$$\begin{array}{r|l} & 32 \\ 3 & 96 \end{array}$$

Phase 3:

Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $96 \div 4$), and be taught to 'carry' the remainder onto the next digit. **If needed, children should use the number line to work out**



$$\begin{array}{r|l} & 18 \\ 4 & 72 \end{array}$$

individual division facts that occur which they are not yet able to recall mentally.

Links to the curriculum:

- ✓ Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- ✓ Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- ✓ Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- ✓ Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- ✓ Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Key Vocabulary: share, share equally, one each, two each, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple.

÷ Division ÷

Year 4

Mental Calculations

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations

Pupils practise mental methods and extend this to three-digit numbers to derive facts.

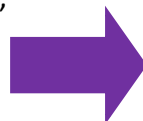
Written Calculations

Divide up to 3-digit numbers by a single digit

$$\begin{array}{r} 218 \\ 4 \overline{) 87^2} 2 \end{array}$$

Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage.

When the answer for the **first column** is zero ($1 \div 5$, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.



$$\begin{array}{r} 037 \\ 5 \overline{) 18^1} 5 \end{array}$$

Include money and measure contexts when confident!

Links to the curriculum:

- ✓ **Recall multiplication and division facts for all numbers up to 12×12 .**
 - ✓ Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
 - ✓ Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
 - ✓ Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ so $600 \div 3 = 200$

- ✓ Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Key Vocabulary:

Share, share equally, one each, two each, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor.

Video explanations:

You will need to log into NCETM website to access these video clips:

[Division - Key Stage 2](#)

Moving to a written algorithm for division - Video 2.1

Representing division with place value counters - Video 2.2

Using place value counters and recording division- Video 2.3

Division with remainders - Video 2.4

Division with exchange - Video 2.5

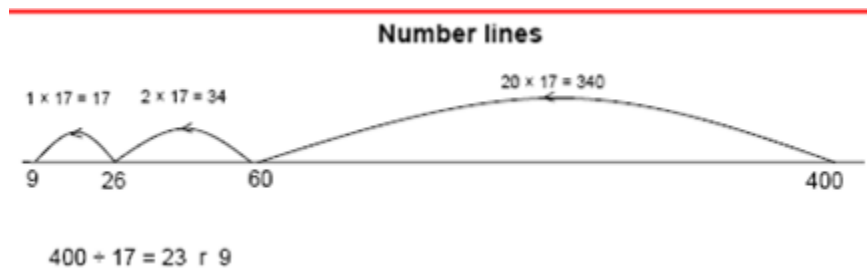
÷ Division ÷

Year 5

Mental Calculations

Pupils should be taught to:

- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- multiply and divide numbers mentally drawing upon known facts identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.



Pupils apply all the multiplication tables and related division facts frequently and use them confidently.

Written Calculations

Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it**, ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.

$$\begin{array}{r} 663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$$

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

The answer to $5309 \div 8$ could be expressed as 663 and five eighths, $663 \text{ r } 5$, as a decimal, or rounded as appropriate to the problem involved.

Include money and measure contexts.

Links to the curriculum:

- ✓ Recall multiplication and division facts for all numbers up to 12×12 (as in Y4).

- ✓ Multiply and divide numbers mentally, drawing upon known facts.
- ✓ Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- ✓ Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- ✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- ✓ Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- ✓ Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- ✓ Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- ✓ Use multiplication and division as inverses.
- ✓ Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).
- ✓ Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Key Vocabulary: share, share equally, one each, two each, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime).

÷ Division ÷

Year 6

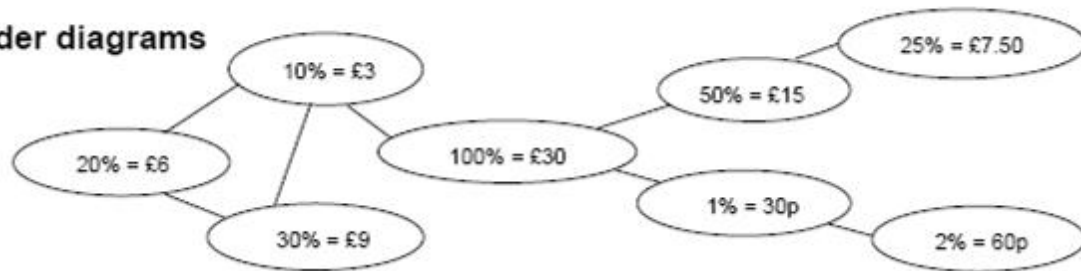
Mental Calculations

Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations.
- identify common factors, common multiples and prime numbers.

I know that 366 will divide by 6 because it has 2 and 3 as factors

Spider diagrams



Solve problems involving addition, subtraction, multiplication and division

- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Written Calculations

Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

Short division, for dividing by a single digit: e.g. $6497 \div 8$

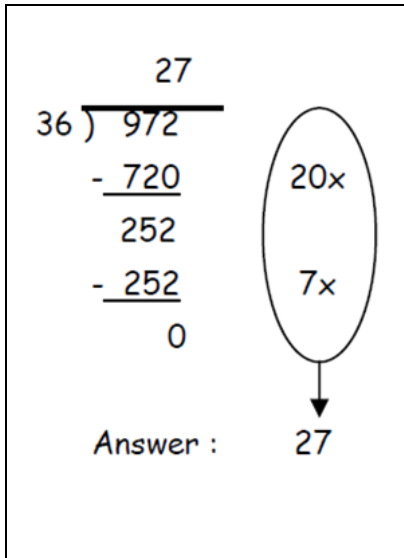
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate

$$\begin{array}{r} 0812.125 \\ 8 \overline{)6497.000} \end{array}$$

degree of accuracy for the problem being solved.

Introduce long division by chunking for dividing by 2 digits.

 <p>27</p> $\begin{array}{r} 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$ <p>Answer : 27</p> <p>20x 7x</p>	<p>Find out 'How many 36s are in 972?' by subtracting 'chunks' of 36, until zero is reached (or until there is a remainder).</p> <ul style="list-style-type: none">Teach pupils to write a 'useful list' first at the side that will help them decide what chunks to use, e.g.: <p>Useful' list:</p> <p>1x = 36 10x = 360 100x = 3600</p> <ul style="list-style-type: none">Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots? Can use 100 lots?' As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their 'useful' lists.
---	--

Links to the curriculum:

- ✓ Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- ✓ Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- ✓ Perform mental calculations, including with mixed operations and large numbers.
- ✓ Identify common factors, common multiples and prime numbers.
- ✓ Solve problems involving all 4 operations.
- ✓ Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- ✓ Use written division methods in cases where the answer has up to two decimal places.
- ✓ Solve problems which require answers to be rounded to specified degrees of accuracy.

Key Vocabulary: share, share equally, one each, two each, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime), common factor.

SIGNED _____

Reviewed : May 2015

Review Date : May 2016